Mathematics
Higher level
Paper 3 – sets, relations and groups

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2218 – 7209

3 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 13]

The binary operation multiplication modulo 10, denoted by $\times_{10}$, is defined on the set $T = \{2, 4, 6, 8\}$ and represented in the following Cayley table.

$\times_{10}$	**2**	**4**	**6**	**8**
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

(a) Show that $\{T, \times_{10}\}$ is a group. (You may assume associativity.) [4]

(b) By making reference to the Cayley table, explain why T is Abelian. [1]

(c) (i) Find the order of each element of $\{T, \times_{10}\}$.

(ii) Hence show that $\{T, \times_{10}\}$ is cyclic and write down all its generators. [6]

The binary operation multiplication modulo 10, denoted by $\times_{10}$, is defined on the set $V = \{1, 3, 5, 7, 9\}$.

(d) Show that $\{V, \times_{10}\}$ is not a group. [2]

2. [Maximum mark: 8]

(a) Consider the sets $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7, 11\}$ and $C = \{1, 3, 7, 15, 31\}$.

(i) Find $(A \cup B) \cap (A \cup C)$.

(ii) Verify that $A \setminus C \neq C \setminus A$. [5]

Let S be a set containing n elements where $n \in \mathbb{N}$.

(b) Show that S has 2^n subsets. [3]

3. [Maximum mark: 8]

The relation R is defined such that xRy if and only if $|x| + |y| = |x + y|$ for $x, y \in \mathbb{R}$.

(a) Show that R is

(i) reflexive;

(ii) symmetric. [4]

(b) Show, by means of an example, that R is not transitive. [4]

4. [Maximum mark: 12]

The set of all permutations of the list of the integers $1, 2, 3, 4$ is a group, S_4, under the operation of function composition.

(a) Determine the order of S_4. [2]

In the group S_4 let $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ and $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$.

(b) Find the proper subgroup H of order 6 containing p_1, p_2 and their compositions. Express each element of H in cycle form. [5]

Let $f: S_4 \to S_4$ be defined by $f(p) = p \circ p$ for $p \in S_4$.

(c) Using p_1 and p_2, explain why f is not a homomorphism. [5]

5. [Maximum mark: 9]

The function $f: \mathbb{Z} \to \mathbb{Z}$ is defined by $f(n) = n + (-1)^n$.

(a) Prove that $f \circ f$ is the identity function. [6]

(b) Show that

(i) f is injective;

(ii) f is surjective. [3]